Market Impacts of Energy Storage in a Transmission-Constrained Power System

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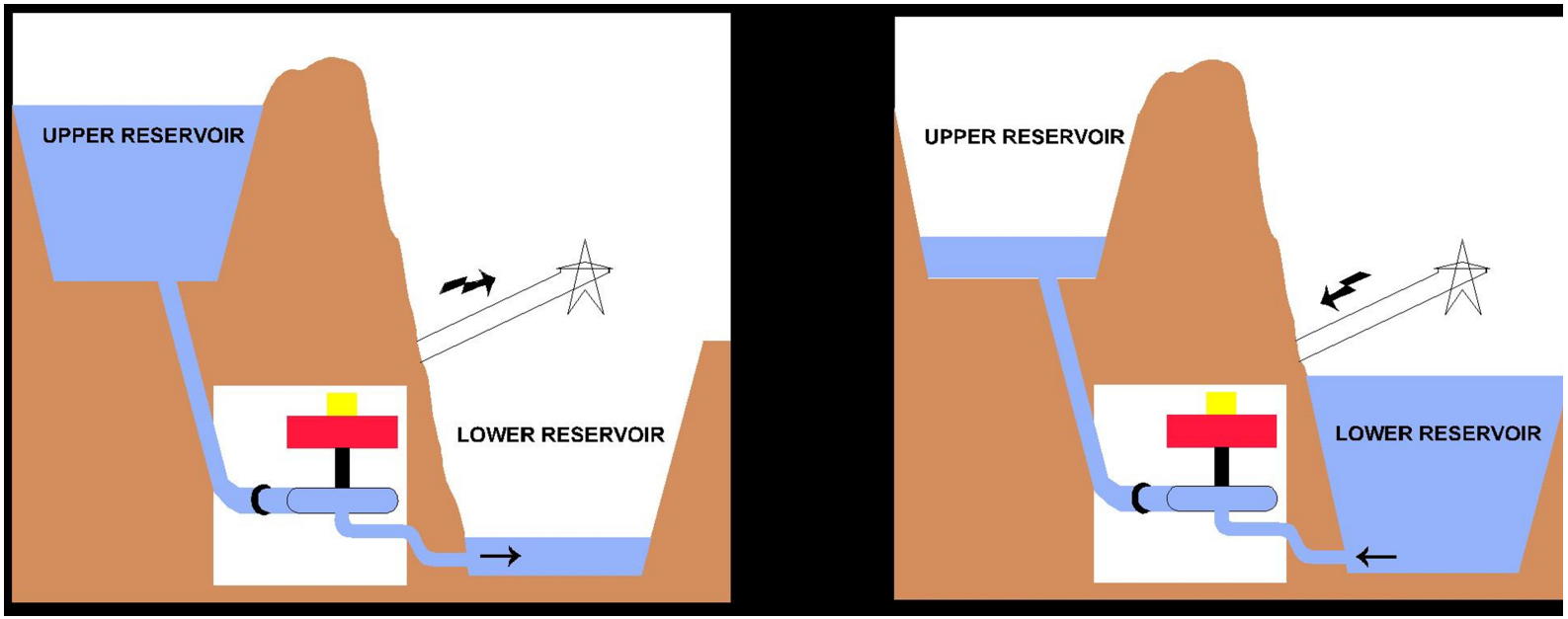
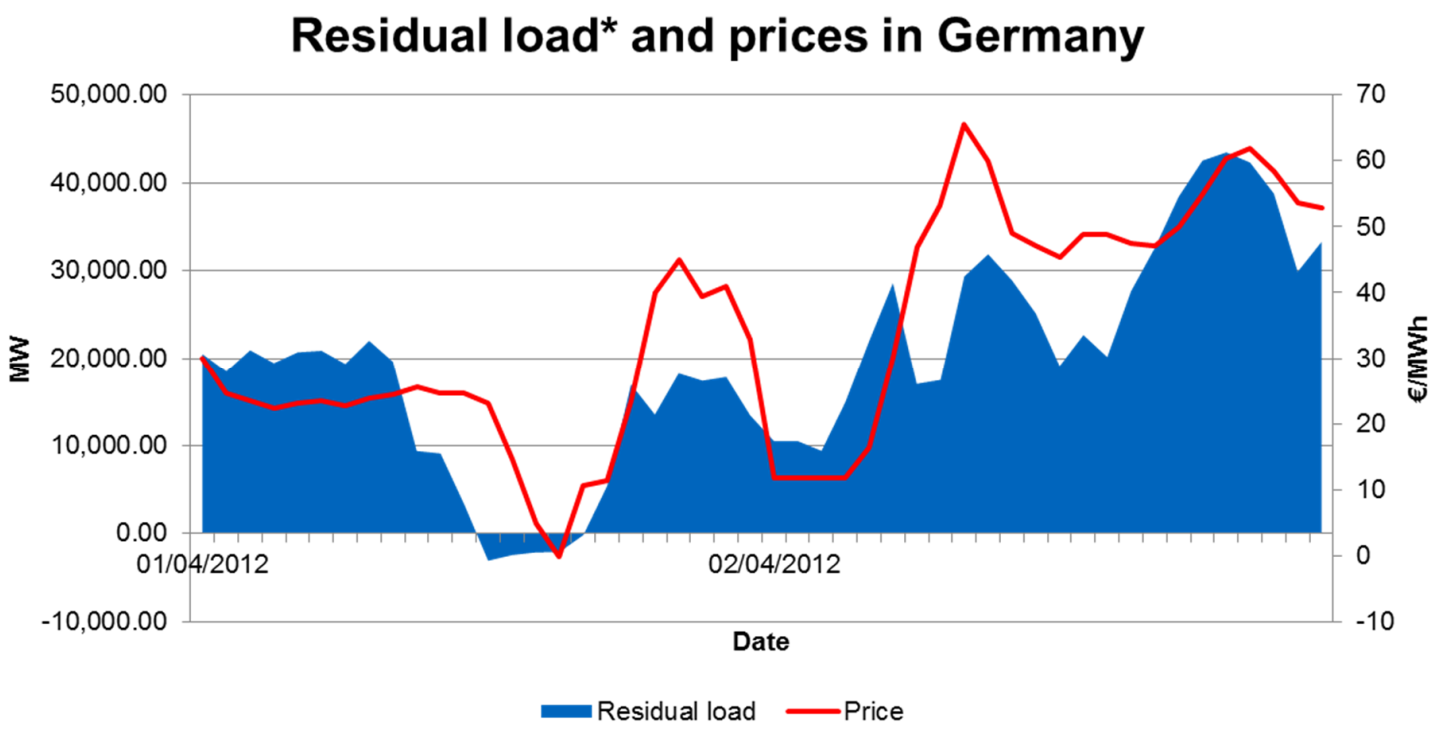
Introduction

"Germany's Effort at Clean Energy Proves Complex," *New York Times*, 18 September 2013

Is Energy Storage the Answer?



Role of Flexible Generation

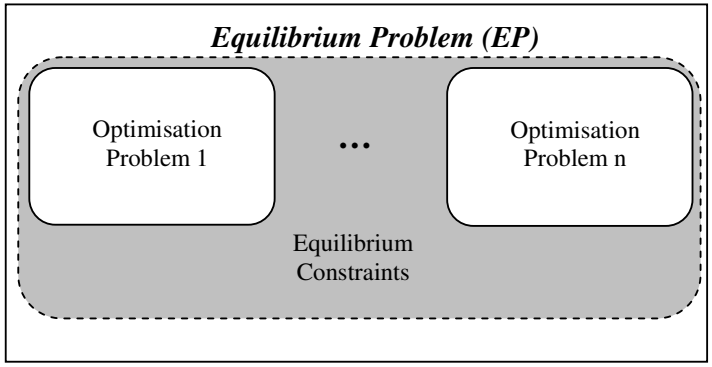


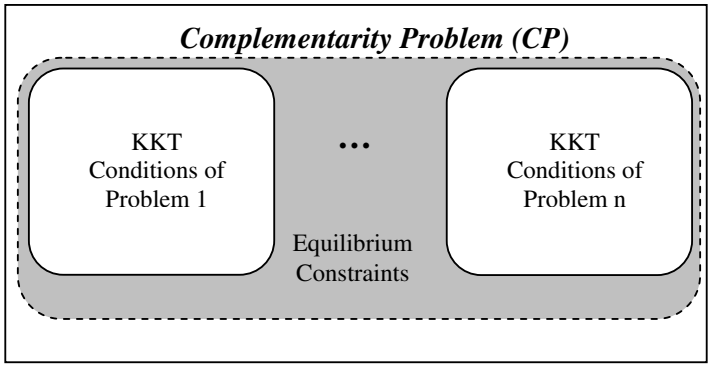
- **Marginal costs: €20-22/MWh (lignite-coal), €30/MWh (CCGT), €45/MWh (gas)**
- **Daily prices: €24.1/MWh and €43.5/MWh**
- ***Residual load = buy volume - wind - solar**

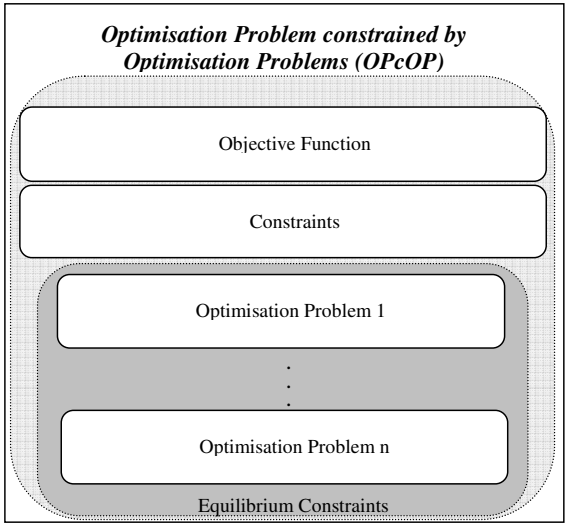
Integration of Renewables and Storage

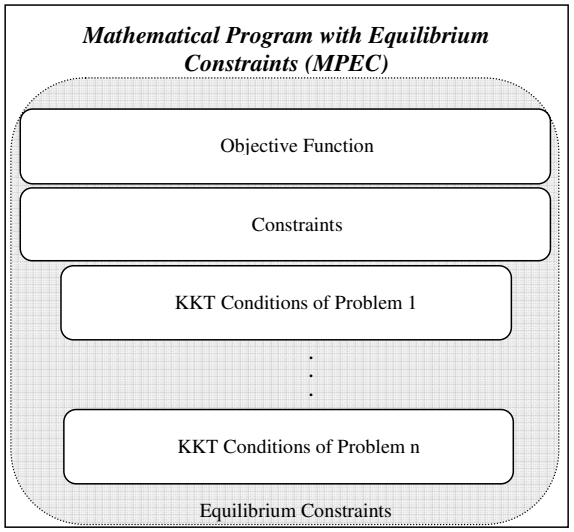
- Hobbs (2001) uses complementarity modelling to analyse strategic behaviour in terms of generation and power flows using a DC network (MCP)
- Kunz (2013) compares current German congestion management with nodal pricing
- Papavasiliou and Oren (2014) examine deferrable demand
- Baringo and Conejo (2012) assume a bi-level problem with a transmission system operator (TSO) constrained by lower-level market clearing (re-cast as MPEC)
- Schill and Kemfert (2011) consider the use of storage in Germany without transmission constraints
- Sioshansi (2014) demonstrates when storage can reduce social welfare or increase GHG emissions (Sioshansi, 2011)
- Transmission-constrained storage papers on optimal siting (Awad et al., 2014) and viability (Lueken and Apt, 2014)

Playing Games









Research Objective and Findings

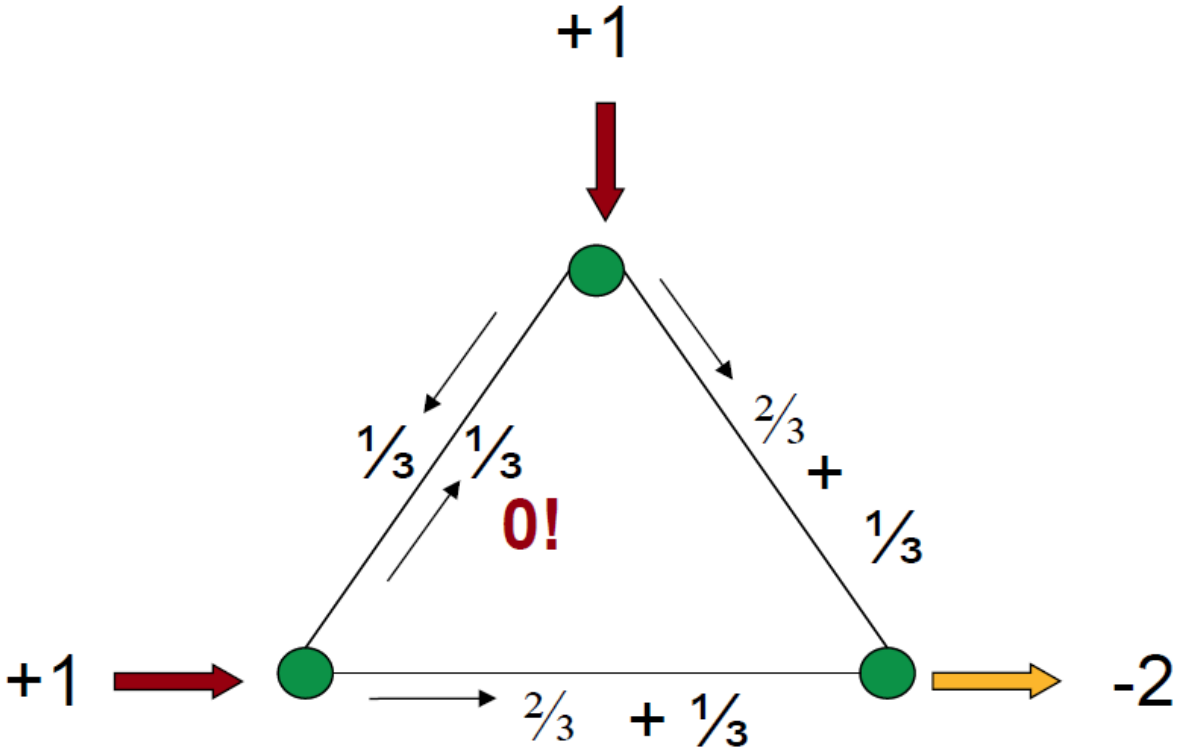
- Examine the system-wide effects of storage given RE output uncertainty, transmission constraints, and market power
- How are prices and GHG emissions affected? What are producers' incentives to use storage?
- Use an MCP to model profit-maximising producers and a grid owner
- Fifteen-node representation of Western Europe calibrated to recent data
- Storage facilitates integration of RE technologies by reducing congestion and ramping
- GHG emissions increase under perfect competition but are unaffected under Cournot oligopoly
- Prevalent transmission flows may be reversed as a result of strategic withholding

Problem Formulation

Assumptions

- Linearised DC power flow
- Stochastic RE output represented by discrete scenarios
- Linear inverse demand: $D^{\text{int}}_{t(s),n} - D^{\text{slp}}_{t(s),n} \sum_{i' \in \mathcal{I}} q_{s,n,i'}$
- Each producer may own both conventional and RE capacity along with storage
  - Maximises expected profit by determining production and sales
  - Generates electricity at node n' and sells it (or stores it) at node n as long as it pays $\omega_{s,n'} - \omega_{s,n}$
  - Generation is constrained by installed capacity
  - Storage must respect minimum and maximum levels as well as charging/discharging rates
- Grid owner
  - Controls transmission flows in order to maximise expected revenues from congestion charges on net imports
  - Must respect thermal limits of transmission lines
- Consider both perfect competition and Cournot oligopoly

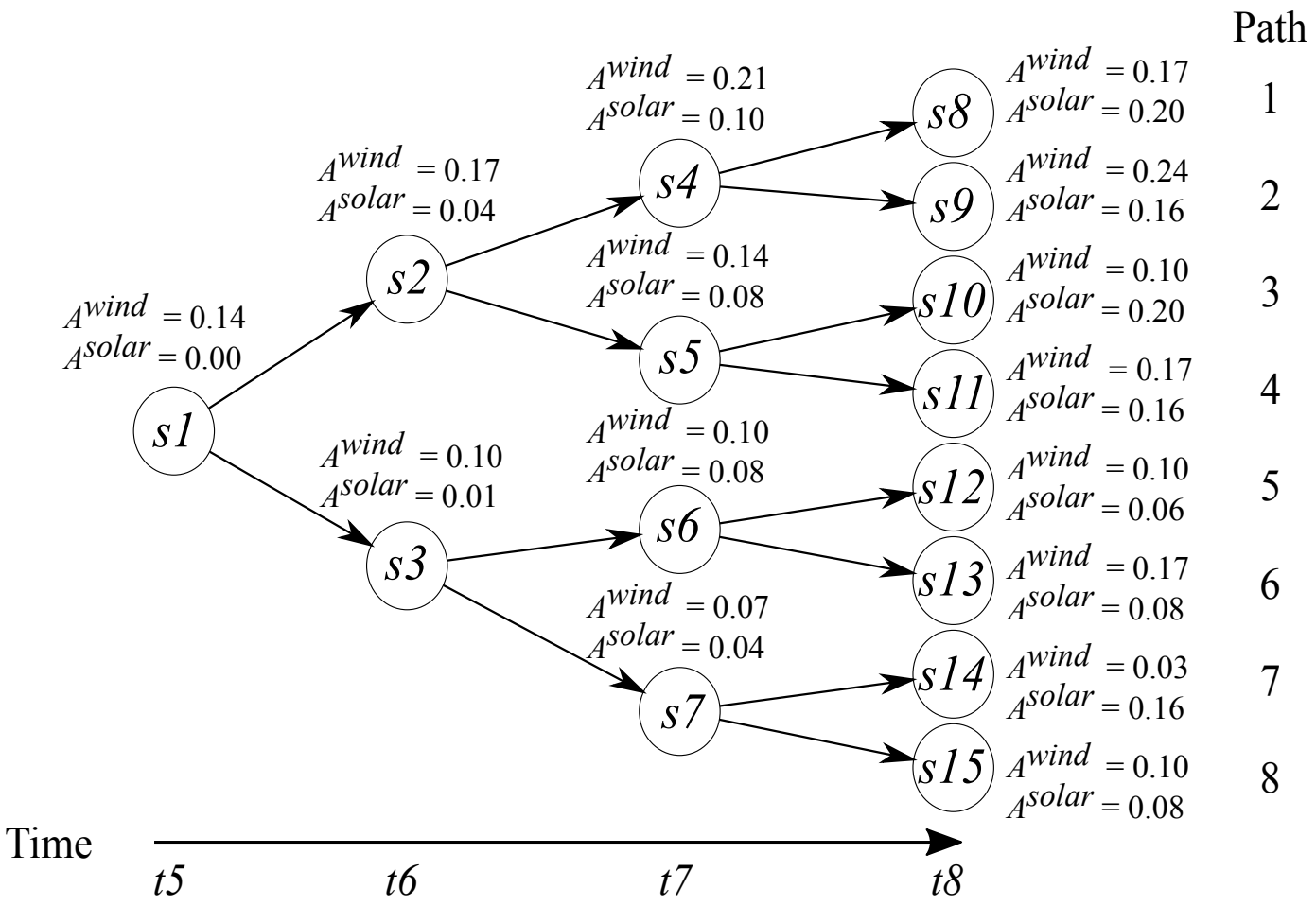
The Path of Least Resistance



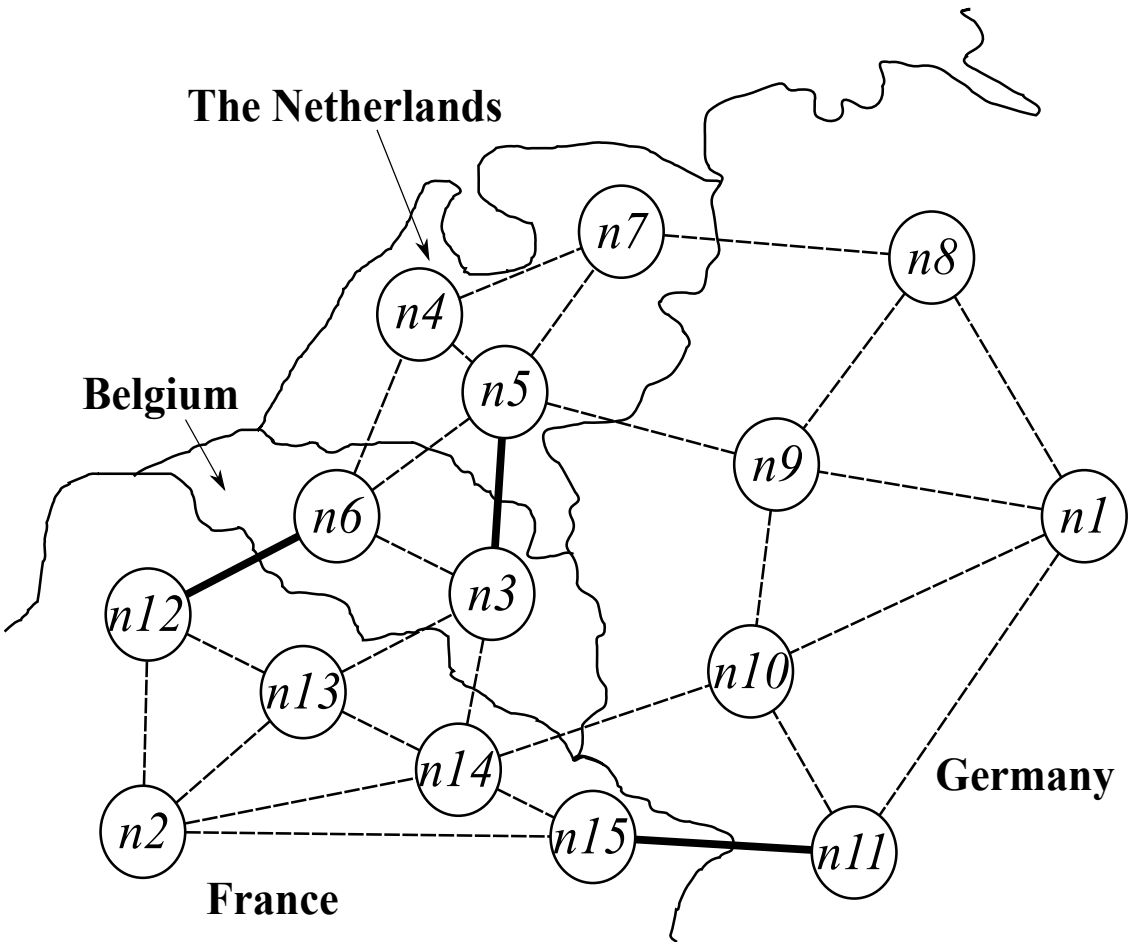
$$\mathit{flow}_\ell = \sum_{n \in \mathcal{N}} H_{\ell,n} v_{s,n}$$

$$\mathit{netimport}_n = \sum_{n' \in \mathcal{N}} T_{t(s)} B_{n',n} v_{s,n'}$$

RE Generation Scenarios



Western European Test Network (Gabriel and Leuthold, 2010)



Producer $i \in \mathcal{I}$

$$\max_{x} \sum_{s\in\mathcal{S}} \sum_{n\in\mathcal{N}} P_s \Bigg[\Big(D^{\text{int}}_{t(s),n} - D^{\text{slp}}_{t(s),n} \sum_{i'\in\mathcal{I}} q_{s,n,i'} - \omega_{s,n} \Big) q_{s,n,i} - \sum_{u\in\mathcal{U}_{n,i}} \big(C^{\text{conv}}_{n,i,u} - \omega_{s,n} \big) g^{\text{conv}}_{s,n,i,u}$$

$$- \sum_{u\in\mathcal{U}_{n,i}} C^{\text{up}}_{n,i,u}\, g^{\text{up}}_{s,n,i,u} + \omega_{s,n} \sum_{e\in\mathcal{E}} g^{e}_{s,n,i} - \omega_{s,n}\, r^{\text{in}}_{s,n,i} - \big(C^{\text{sto}} - \omega_{s,n} \big) r^{\text{out}}_{s,n,i} \Bigg] \tag{1}$$

$$\text{s.t.} \sum_{n\in\mathcal{N}} q_{s,n,i} = \sum_{n\in\mathcal{N}} \sum_{u\in\mathcal{U}_{n,i}} g^{\text{conv}}_{s,n,i,u} + \sum_{n\in\mathcal{N}} \sum_{e\in\mathcal{E}} g^{e}_{s,n,i} + \sum_{n\in\mathcal{N}} r^{\text{out}}_{s,n,i} - \sum_{n\in\mathcal{N}} r^{\text{in}}_{s,n,i} \; (\theta_{s,i}), \; \forall s \tag{2}$$

$$g^{\text{conv}}_{s,n,i,u} \leq T_{t(s)}\, \overline{G}^{\text{conv}}_{n,i,u} \; (\beta^{\text{conv}}_{s,n,i,u}), \; \forall n, s, \forall u \in \mathcal{U}_{n,i} \tag{3}$$

$$g^{\text{conv}}_{s,n,i,u} - g^{\text{conv}}_{a(s),n,i,u} \leq g^{\text{up}}_{s,n,i,u} \; (\beta^{\text{up}}_{s,n,i,u}), \; \forall n, s, \forall u \in \mathcal{U}_{n,i} \tag{4}$$

$$g^{e}_{s,n,i} = T_{t(s)}\, A^{e}_{s,n}\, \overline{G}^{e}_{n,i} \; (\beta^{e}_{s,n,i}), \; \forall e, n, s \tag{5}$$

$$r_{s,n,i} = T_{t(s)}\, (1 - E^{\text{sto}})\, r_{a(s),n,i} + E^{\text{in}}\, r^{\text{in}}_{s,n,i} - r^{\text{out}}_{s,n,i} \; (\lambda^{\text{bal}}_{s,n,i}), \; \forall n, s \tag{6}$$

$$r^{\text{in}}_{s,n,i} \leq T_{t(s)}\, R^{\text{in}}\, \overline{R}_{n,i} \; (\lambda^{\text{in}}_{s,n,i}), \; \forall n, s \tag{7}$$

$$r^{\text{out}}_{s,n,i} \leq T_{t(s)}\, R^{\text{out}}\, \overline{R}_{n,i} \; (\lambda^{\text{out}}_{s,n,i}), \; \forall n, s \tag{8}$$

$$\underline{R}_{n,i} \leq r_{s,n,i} \leq \overline{R}_{n,i} \; (\lambda^{\text{lb}}_{s,n,i}, \lambda^{\text{ub}}_{s,n,i}), \; \forall n, s \tag{9}$$

$$g^{\text{conv}}_{s,n,i,u}, g^{\text{up}}_{s,n,i,u} \geq 0, \; \forall n, s, \forall u \in \mathcal{U}_{n,i}; \; g^{e}_{s,n,i} \geq 0, \; \forall e, n, s; \; q_{s,n,i}, r_{s,n,i}, r^{\text{in}}_{s,n,i}, r^{\text{out}}_{s,n,i} \geq 0, \; \forall n, s \tag{10}$$

$$x := \{ g^{\text{conv}}_{s,n,i,u}, g^{e}_{s,n,i}, g^{\text{up}}_{s,n,i,u}, q_{s,n,i}, r_{s,n,i}, r^{\text{in}}_{s,n,i}, r^{\text{out}}_{s,n,i} : e \in \mathcal{E}, i \in \mathcal{I}, n \in \mathcal{N}, s \in \mathcal{S}, u \in \mathcal{U}_{n,i} \}$$

Producer $i \in \mathcal{I}$

$$\max_{x} \sum_{s\in\mathcal{S}} \sum_{n\in\mathcal{N}} P_s \Bigg[\Big(D^{\text{int}}_{t(s),n} - D^{\text{slp}}_{t(s),n} \sum_{i'\in\mathcal{I}} q_{s,n,i'} - \omega_{s,n} \Big) q_{s,n,i} - \sum_{u\in\mathcal{U}_{n,i}} \big(C^{\text{conv}}_{n,i,u} - \omega_{s,n} \big) g^{\text{conv}}_{s,n,i,u}$$

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$$g^{\text{conv}}_{s,n,i,u} \leq T_{t(s)}\, \overline{G}^{\text{conv}}_{n,i,u} \ (\beta^{\text{conv}}_{s,n,i,u}), \ \forall n, s, \forall u \in \mathcal{U}_{n,i} \tag{3}$$

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$$- \sum_{u\in\mathcal{U}_{n,i}} C^{\text{up}}_{n,i,u}\, g^{\text{up}}_{s,n,i,u} + \omega_{s,n} \sum_{e\in\mathcal{E}} g^{e}_{s,n,i} - \omega_{s,n}\, r^{\text{in}}_{s,n,i} - \big(C^{\text{sto}} - \omega_{s,n} \big) r^{\text{out}}_{s,n,i} \Bigg] \tag{1}$$

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$$g^{\text{conv}}_{s,n,i,u} \leq T_{t(s)}\, \overline{G}^{\text{conv}}_{n,i,u} \; (\beta^{\text{conv}}_{s,n,i,u}), \; \forall n, s, \forall u \in \mathcal{U}_{n,i} \tag{3}$$

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$$r^{\text{in}}_{s,n,i} \leq T_{t(s)}\, R^{\text{in}}\, \overline{R}_{n,i} \; (\lambda^{\text{in}}_{s,n,i}), \; \forall n, s \tag{7}$$

$$r^{\text{out}}_{s,n,i} \leq T_{t(s)}\, R^{\text{out}}\, \overline{R}_{n,i} \; (\lambda^{\text{out}}_{s,n,i}), \; \forall n, s \tag{8}$$

$$\underline{R}_{n,i} \leq r_{s,n,i} \leq \overline{R}_{n,i} \; (\lambda^{\text{lb}}_{s,n,i}, \lambda^{\text{ub}}_{s,n,i}), \; \forall n, s \tag{9}$$

$$g^{\text{conv}}_{s,n,i,u}, g^{\text{up}}_{s,n,i,u} \geq 0, \; \forall n, s, \forall u \in \mathcal{U}_{n,i}; \; g^{e}_{s,n,i} \geq 0, \; \forall e, n, s; \; q_{s,n,i}, r_{s,n,i}, r^{\text{in}}_{s,n,i}, r^{\text{out}}_{s,n,i} \geq 0, \; \forall n, s \tag{10}$$

$$x := \{ g^{\text{conv}}_{s,n,i,u}, g^{e}_{s,n,i}, g^{\text{up}}_{s,n,i,u}, q_{s,n,i}, r_{s,n,i}, r^{\text{in}}_{s,n,i}, r^{\text{out}}_{s,n,i} : e \in \mathcal{E}, i \in \mathcal{I}, n \in \mathcal{N}, s \in \mathcal{S}, u \in \mathcal{U}_{n,i} \}$$

Producer $i \in \mathcal{I}$

$$\max_x \sum_{s\in\mathcal{S}} \sum_{n\in\mathcal{N}} P_s \Bigg[\Big(D^{\text{int}}_{t(s),n} - D^{\text{slp}}_{t(s),n} \sum_{i'\in\mathcal{I}} q_{s,n,i'} - \omega_{s,n} \Big) q_{s,n,i} - \sum_{u\in\mathcal{U}_{n,i}} \big(C^{\text{conv}}_{n,i,u} - \omega_{s,n} \big) g^{\text{conv}}_{s,n,i,u}$$

$$- \sum_{u\in\mathcal{U}_{n,i}} C^{\text{up}}_{n,i,u}\, g^{\text{up}}_{s,n,i,u} + \omega_{s,n} \sum_{e\in\mathcal{E}} g^{e}_{s,n,i} - \omega_{s,n}\, r^{\text{in}}_{s,n,i} - \big(C^{\text{sto}} - \omega_{s,n} \big) r^{\text{out}}_{s,n,i} \Bigg] \tag{1}$$

$$\text{s.t.} \sum_{n\in\mathcal{N}} q_{s,n,i} = \sum_{n\in\mathcal{N}} \sum_{u\in\mathcal{U}_{n,i}} g^{\text{conv}}_{s,n,i,u} + \sum_{n\in\mathcal{N}} \sum_{e\in\mathcal{E}} g^{e}_{s,n,i} + \sum_{n\in\mathcal{N}} r^{\text{out}}_{s,n,i} - \sum_{n\in\mathcal{N}} r^{\text{in}}_{s,n,i} \ (\theta_{s,i}), \ \forall s \tag{2}$$

$$g^{\text{conv}}_{s,n,i,u} \le T_{t(s)}\, \overline{G}^{\text{conv}}_{n,i,u} \ (\beta^{\text{conv}}_{s,n,i,u}), \ \forall n, s, \forall u \in \mathcal{U}_{n,i} \tag{3}$$

$$g^{\text{conv}}_{s,n,i,u} - g^{\text{conv}}_{a(s),n,i,u} \le g^{\text{up}}_{s,n,i,u} \ (\beta^{\text{up}}_{s,n,i,u}), \ \forall n, s, \forall u \in \mathcal{U}_{n,i} \tag{4}$$

$$g^{e}_{s,n,i} = T_{t(s)}\, A^{e}_{s,n}\, \overline{G}^{e}_{n,i} \ (\beta^{e}_{s,n,i}), \ \forall e, n, s \tag{5}$$

$$r_{s,n,i} = T_{t(s)} \, (1 - E^{\text{sto}})\, r_{a(s),n,i} + E^{\text{in}}\, r^{\text{in}}_{s,n,i} - r^{\text{out}}_{s,n,i} \ (\lambda^{\text{bal}}_{s,n,i}), \ \forall n, s \tag{6}$$

$$r^{\text{in}}_{s,n,i} \le T_{t(s)}\, R^{\text{in}}\, \overline{R}_{n,i} \ (\lambda^{\text{in}}_{s,n,i}), \ \forall n, s \tag{7}$$

$$r^{\text{out}}_{s,n,i} \le T_{t(s)}\, R^{\text{out}}\, \overline{R}_{n,i} \ (\lambda^{\text{out}}_{s,n,i}), \ \forall n, s \tag{8}$$

$$\underline{R}_{n,i} \le r_{s,n,i} \le \overline{R}_{n,i} \ (\lambda^{\text{lb}}_{s,n,i}, \lambda^{\text{ub}}_{s,n,i}), \ \forall n, s \tag{9}$$

$$g^{\text{conv}}_{s,n,i,u}, g^{\text{up}}_{s,n,i,u} \ge 0, \ \forall n, s, \forall u \in \mathcal{U}_{n,i}; \ g^{e}_{s,n,i} \ge 0, \ \forall e, n, s; \ q_{s,n,i}, r_{s,n,i}, r^{\text{in}}_{s,n,i}, r^{\text{out}}_{s,n,i} \ge 0, \ \forall n, s \tag{10}$$

$$x := \{ g^{\text{conv}}_{s,n,i,u}, g^{e}_{s,n,i}, g^{\text{up}}_{s,n,i,u}, q_{s,n,i}, r_{s,n,i}, r^{\text{in}}_{s,n,i}, r^{\text{out}}_{s,n,i} : e \in \mathcal{E}, i \in \mathcal{I}, n \in \mathcal{N}, s \in \mathcal{S}, u \in \mathcal{U}_{n,i} \}$$

Producer $i \in \mathcal{I}$

$$\max_{x} \sum_{s\in\mathcal{S}} \sum_{n\in\mathcal{N}} P_s \Bigg[\Big(D^{\text{int}}_{t(s),n} - D^{\text{slp}}_{t(s),n} \sum_{i'\in\mathcal{I}} q_{s,n,i'} - \omega_{s,n} \Big) q_{s,n,i} - \sum_{u\in\mathcal{U}_{n,i}} \big(C^{\text{conv}}_{n,i,u} - \omega_{s,n} \big) g^{\text{conv}}_{s,n,i,u}$$

$$- \sum_{u\in\mathcal{U}_{n,i}} C^{\text{up}}_{n,i,u}\, g^{\text{up}}_{s,n,i,u} + \omega_{s,n} \sum_{e\in\mathcal{E}} g^{e}_{s,n,i} - \omega_{s,n}\, r^{\text{in}}_{s,n,i} - \big(C^{\text{sto}} - \omega_{s,n} \big) r^{\text{out}}_{s,n,i} \Bigg] \tag{1}$$

$$\text{s.t.} \sum_{n\in\mathcal{N}} q_{s,n,i} = \sum_{n\in\mathcal{N}} \sum_{u\in\mathcal{U}_{n,i}} g^{\text{conv}}_{s,n,i,u} + \sum_{n\in\mathcal{N}} \sum_{e\in\mathcal{E}} g^{e}_{s,n,i} + \sum_{n\in\mathcal{N}} r^{\text{out}}_{s,n,i} - \sum_{n\in\mathcal{N}} r^{\text{in}}_{s,n,i} \ (\theta_{s,i}), \ \forall s \tag{2}$$

$$g^{\text{conv}}_{s,n,i,u} \leq T_{t(s)}\, \overline{G}^{\text{conv}}_{n,i,u} \ (\beta^{\text{conv}}_{s,n,i,u}), \ \forall n, s, \forall u \in \mathcal{U}_{n,i} \tag{3}$$

$$g^{\text{conv}}_{s,n,i,u} - g^{\text{conv}}_{a(s),n,i,u} \leq g^{\text{up}}_{s,n,i,u} \ (\beta^{\text{up}}_{s,n,i,u}), \ \forall n, s, \forall u \in \mathcal{U}_{n,i} \tag{4}$$

$$g^{e}_{s,n,i} = T_{t(s)}\, A^{e}_{s,n}\, \overline{G}^{e}_{n,i} \ (\beta^{e}_{s,n,i}), \ \forall e, n, s \tag{5}$$

$$r_{s,n,i} = T_{t(s)}\, (1 - E^{\text{sto}})\, r_{a(s),n,i} + E^{\text{in}}\, r^{\text{in}}_{s,n,i} - r^{\text{out}}_{s,n,i} \ (\lambda^{\text{bal}}_{s,n,i}), \ \forall n, s \tag{6}$$

$$r^{\text{in}}_{s,n,i} \leq T_{t(s)}\, R^{\text{in}}\, \overline{R}_{n,i} \ (\lambda^{\text{in}}_{s,n,i}), \ \forall n, s \tag{7}$$

$$r^{\text{out}}_{s,n,i} \leq T_{t(s)}\, R^{\text{out}}\, \overline{R}_{n,i} \ (\lambda^{\text{out}}_{s,n,i}), \ \forall n, s \tag{8}$$

$$\underline{R}_{n,i} \leq r_{s,n,i} \leq \overline{R}_{n,i} \ (\lambda^{\text{lb}}_{s,n,i}, \lambda^{\text{ub}}_{s,n,i}), \ \forall n, s \tag{9}$$

$$g^{\text{conv}}_{s,n,i,u}, g^{\text{up}}_{s,n,i,u} \geq 0, \ \forall n, s, \forall u \in \mathcal{U}_{n,i}; \ g^{e}_{s,n,i} \geq 0, \ \forall e, n, s; \ q_{s,n,i}, r_{s,n,i}, r^{\text{in}}_{s,n,i}, r^{\text{out}}_{s,n,i} \geq 0, \ \forall n, s \tag{10}$$

$$x := \{ g^{\text{conv}}_{s,n,i,u}, g^{e}_{s,n,i}, g^{\text{up}}_{s,n,i,u}, q_{s,n,i}, r_{s,n,i}, r^{\text{in}}_{s,n,i}, r^{\text{out}}_{s,n,i} : e \in \mathcal{E}, i \in \mathcal{I}, n \in \mathcal{N}, s \in \mathcal{S}, u \in \mathcal{U}_{n,i} \}$$

Producer $i \in \mathcal{I}$

$$\max_{x} \sum_{s\in\mathcal{S}} \sum_{n\in\mathcal{N}} P_s \Bigg[\Big(D^{\text{int}}_{t(s),n} - D^{\text{slp}}_{t(s),n} \sum_{i'\in\mathcal{I}} q_{s,n,i'} - \omega_{s,n} \Big) q_{s,n,i} - \sum_{u\in\mathcal{U}_{n,i}} \big(C^{\text{conv}}_{n,i,u} - \omega_{s,n} \big) g^{\text{conv}}_{s,n,i,u}$$

$$- \sum_{u\in\mathcal{U}_{n,i}} C^{\text{up}}_{n,i,u}\, g^{\text{up}}_{s,n,i,u} + \omega_{s,n} \sum_{e\in\mathcal{E}} g^{e}_{s,n,i} - \omega_{s,n}\, r^{\text{in}}_{s,n,i} - \big(C^{\text{sto}} - \omega_{s,n} \big) r^{\text{out}}_{s,n,i} \Bigg] \tag{1}$$

$$\text{s.t.} \sum_{n\in\mathcal{N}} q_{s,n,i} = \sum_{n\in\mathcal{N}} \sum_{u\in\mathcal{U}_{n,i}} g^{\text{conv}}_{s,n,i,u} + \sum_{n\in\mathcal{N}} \sum_{e\in\mathcal{E}} g^{e}_{s,n,i} + \sum_{n\in\mathcal{N}} r^{\text{out}}_{s,n,i} - \sum_{n\in\mathcal{N}} r^{\text{in}}_{s,n,i} \ (\theta_{s,i}), \ \forall s \tag{2}$$

$$g^{\text{conv}}_{s,n,i,u} \leq T_{t(s)}\, \overline{G}^{\text{conv}}_{n,i,u} \ (\beta^{\text{conv}}_{s,n,i,u}), \ \forall n, s, \forall u \in \mathcal{U}_{n,i} \tag{3}$$

$$g^{\text{conv}}_{s,n,i,u} - g^{\text{conv}}_{a(s),n,i,u} \leq g^{\text{up}}_{s,n,i,u} \ (\beta^{\text{up}}_{s,n,i,u}), \ \forall n, s, \forall u \in \mathcal{U}_{n,i} \tag{4}$$

$$g^{e}_{s,n,i} = T_{t(s)}\, A^{e}_{s,n}\, \overline{G}^{e}_{n,i} \ (\beta^{e}_{s,n,i}), \ \forall e, n, s \tag{5}$$

$$r_{s,n,i} = T_{t(s)}\, (1 - E^{\text{sto}})\, r_{a(s),n,i} + E^{\text{in}}\, r^{\text{in}}_{s,n,i} - r^{\text{out}}_{s,n,i} \ (\lambda^{\text{bal}}_{s,n,i}), \ \forall n, s \tag{6}$$

$$r^{\text{in}}_{s,n,i} \leq T_{t(s)}\, R^{\text{in}}\, \overline{R}_{n,i} \ (\lambda^{\text{in}}_{s,n,i}), \ \forall n, s \tag{7}$$

$$r^{\text{out}}_{s,n,i} \leq T_{t(s)}\, R^{\text{out}}\, \overline{R}_{n,i} \ (\lambda^{\text{out}}_{s,n,i}), \ \forall n, s \tag{8}$$

$$\underline{R}_{n,i} \leq r_{s,n,i} \leq \overline{R}_{n,i} \ (\lambda^{\text{lb}}_{s,n,i}, \lambda^{\text{ub}}_{s,n,i}), \ \forall n, s \tag{9}$$

$$g^{\text{conv}}_{s,n,i,u}, g^{\text{up}}_{s,n,i,u} \geq 0, \ \forall n, s, \forall u \in \mathcal{U}_{n,i}; \ g^{e}_{s,n,i} \geq 0, \ \forall e, n, s; \ q_{s,n,i}, r_{s,n,i}, r^{\text{in}}_{s,n,i}, r^{\text{out}}_{s,n,i} \geq 0, \ \forall n, s \tag{10}$$

$$x := \{ g^{\text{conv}}_{s,n,i,u}, g^{e}_{s,n,i}, g^{\text{up}}_{s,n,i,u}, q_{s,n,i}, r_{s,n,i}, r^{\text{in}}_{s,n,i}, r^{\text{out}}_{s,n,i} : e \in \mathcal{E}, i \in \mathcal{I}, n \in \mathcal{N}, s \in \mathcal{S}, u \in \mathcal{U}_{n,i} \}$$

Producer $i \in \mathcal{I}$

$$\max_x \sum_{s\in\mathcal{S}} \sum_{n\in\mathcal{N}} P_s \Bigg[\Big(D^{\text{int}}_{t(s),n} - D^{\text{slp}}_{t(s),n} \sum_{i'\in\mathcal{I}} q_{s,n,i'} - \omega_{s,n} \Big) q_{s,n,i} - \sum_{u\in\mathcal{U}_{n,i}} \big(C^{\text{conv}}_{n,i,u} - \omega_{s,n} \big) g^{\text{conv}}_{s,n,i,u}$$

$$- \sum_{u\in\mathcal{U}_{n,i}} C^{\text{up}}_{n,i,u}\, g^{\text{up}}_{s,n,i,u} + \omega_{s,n} \sum_{e\in\mathcal{E}} g^{e}_{s,n,i} - \omega_{s,n}\, r^{\text{in}}_{s,n,i} - \big(C^{\text{sto}} - \omega_{s,n} \big) r^{\text{out}}_{s,n,i} \Bigg] \tag{1}$$

$$\text{s.t.} \sum_{n\in\mathcal{N}} q_{s,n,i} = \sum_{n\in\mathcal{N}} \sum_{u\in\mathcal{U}_{n,i}} g^{\text{conv}}_{s,n,i,u} + \sum_{n\in\mathcal{N}} \sum_{e\in\mathcal{E}} g^{e}_{s,n,i} + \sum_{n\in\mathcal{N}} r^{\text{out}}_{s,n,i} - \sum_{n\in\mathcal{N}} r^{\text{in}}_{s,n,i} \; (\theta_{s,i}), \; \forall s \tag{2}$$

$$g^{\text{conv}}_{s,n,i,u} \le T_{t(s)}\, \overline{G}^{\text{conv}}_{n,i,u} \; (\beta^{\text{conv}}_{s,n,i,u}), \; \forall n, s, \forall u \in \mathcal{U}_{n,i} \tag{3}$$

$$g^{\text{conv}}_{s,n,i,u} - g^{\text{conv}}_{a(s),n,i,u} \le g^{\text{up}}_{s,n,i,u} \; (\beta^{\text{up}}_{s,n,i,u}), \; \forall n, s, \forall u \in \mathcal{U}_{n,i} \tag{4}$$

$$g^{e}_{s,n,i} = T_{t(s)}\, A^{e}_{s,n}\, \overline{G}^{e}_{n,i} \; (\beta^{e}_{s,n,i}), \; \forall e, n, s \tag{5}$$

$$r_{s,n,i} = T_{t(s)}\, (1 - E^{\text{sto}})\, r_{a(s),n,i} + E^{\text{in}}\, r^{\text{in}}_{s,n,i} - r^{\text{out}}_{s,n,i} \; (\lambda^{\text{bal}}_{s,n,i}), \; \forall n, s \tag{6}$$

$$r^{\text{in}}_{s,n,i} \le T_{t(s)}\, R^{\text{in}}\, \overline{R}_{n,i} \; (\lambda^{\text{in}}_{s,n,i}), \; \forall n, s \tag{7}$$

$$r^{\text{out}}_{s,n,i} \le T_{t(s)}\, R^{\text{out}}\, \overline{R}_{n,i} \; (\lambda^{\text{out}}_{s,n,i}), \; \forall n, s \tag{8}$$

$$\underline{R}_{n,i} \le r_{s,n,i} \le \overline{R}_{n,i} \; (\lambda^{\text{lb}}_{s,n,i}, \lambda^{\text{ub}}_{s,n,i}), \; \forall n, s \tag{9}$$

$$g^{\text{conv}}_{s,n,i,u}, g^{\text{up}}_{s,n,i,u} \ge 0, \; \forall n, s, \forall u \in \mathcal{U}_{n,i}; \; g^{e}_{s,n,i} \ge 0, \; \forall e, n, s; \; q_{s,n,i}, r_{s,n,i}, r^{\text{in}}_{s,n,i}, r^{\text{out}}_{s,n,i} \ge 0, \; \forall n, s \tag{10}$$

$$x := \{ g^{\text{conv}}_{s,n,i,u}, g^{e}_{s,n,i}, g^{\text{up}}_{s,n,i,u}, q_{s,n,i}, r_{s,n,i}, r^{\text{in}}_{s,n,i}, r^{\text{out}}_{s,n,i} : e \in \mathcal{E}, i \in \mathcal{I}, n \in \mathcal{N}, s \in \mathcal{S}, u \in \mathcal{U}_{n,i} \}$$

Grid Owner and Market Clearing

$$\max_{v_{s,n}} \sum_{s\in\mathcal{S}} \sum_{n\in\mathcal{N}} P_s\, \omega_{s,n} \sum_{n'\in\mathcal{N}} T_{t(s)}\, B_{n,n'}\, v_{s,n'} \tag{11}$$

$$\text{s.t.} \sum_{n\in\mathcal{N}} H_{\ell,n} v_{s,n} \leq K_\ell\ (\overline{\mu}_{s,\ell}),\ \forall \ell, s \tag{12}$$

$$-\sum_{n\in\mathcal{N}} H_{\ell,n} v_{s,n} \leq K_\ell\ (\underline{\mu}_{s,\ell}),\ \forall \ell, s \tag{13}$$

$$S_n\, v_{s,n} = 0\ (\gamma_{s,n}),\ \forall n, s \tag{14}$$

$$v_{s,n} \text{ u.r.s.},\ \forall n, s \tag{15}$$

$$\sum_{i\in\mathcal{I}} q_{s,n,i} - \sum_{i\in\mathcal{I}} \sum_{u\in\mathcal{U}_{n,i}} g^{\text{conv}}_{s,n,i,u} - \sum_{i\in\mathcal{I}} \sum_{e\in\mathcal{E}} g^{e}_{s,n,i} - \sum_{i\in\mathcal{I}} r^{\text{out}}_{s,n,i}$$
$$+ \sum_{i\in\mathcal{I}} r^{\text{in}}_{s,n,i} - \sum_{n'\in\mathcal{N}} T_{t(s)} B_{n,n'} v_{s,n'} = 0, \text{with } \omega_{s,n} \text{ u.r.s.},\ \forall n, s \tag{16}$$

Grid Owner and Market Clearing

$$\max_{v_{s,n}} \sum_{s\in\mathcal{S}} \sum_{n\in\mathcal{N}} P_s\, \omega_{s,n} \sum_{n'\in\mathcal{N}} T_{t(s)}\, B_{n,n'}\, v_{s,n'} \tag{11}$$

$$\text{s.t.} \sum_{n\in\mathcal{N}} H_{\ell,n} v_{s,n} \leq K_\ell\ (\overline{\mu}_{s,\ell}),\ \forall \ell, s \tag{12}$$

$$-\sum_{n\in\mathcal{N}} H_{\ell,n} v_{s,n} \leq K_\ell\ (\underline{\mu}_{s,\ell}),\ \forall \ell, s \tag{13}$$

$$S_n\, v_{s,n} = 0\ (\gamma_{s,n}),\ \forall n, s \tag{14}$$

$$v_{s,n} \text{ u.r.s.},\ \forall n, s \tag{15}$$

$$\sum_{i\in\mathcal{I}} q_{s,n,i} - \sum_{i\in\mathcal{I}} \sum_{u\in\mathcal{U}_{n,i}} g^{\text{conv}}_{s,n,i,u} - \sum_{i\in\mathcal{I}} \sum_{e\in\mathcal{E}} g^{e}_{s,n,i} - \sum_{i\in\mathcal{I}} r^{\text{out}}_{s,n,i}$$
$$+ \sum_{i\in\mathcal{I}} r^{\text{in}}_{s,n,i} - \sum_{n'\in\mathcal{N}} T_{t(s)} B_{n,n'} v_{s,n'} = 0, \text{with } \omega_{s,n} \text{ u.r.s.},\ \forall n, s \tag{16}$$

Grid Owner and Market Clearing

$$\max_{v_{s,n}} \sum_{s\in\mathcal{S}} \sum_{n\in\mathcal{N}} P_s\, \omega_{s,n} \sum_{n'\in\mathcal{N}} T_{t(s)}\, B_{n,n'}\, v_{s,n'} \tag{11}$$

$$\text{s.t.} \sum_{n\in\mathcal{N}} H_{\ell,n} v_{s,n} \leq K_\ell\ (\overline{\mu}_{s,\ell}),\ \forall \ell, s \tag{12}$$

$$-\sum_{n\in\mathcal{N}} H_{\ell,n} v_{s,n} \leq K_\ell\ (\underline{\mu}_{s,\ell}),\ \forall \ell, s \tag{13}$$

$$S_n\, v_{s,n} = 0\ (\gamma_{s,n}),\ \forall n, s \tag{14}$$

$$v_{s,n} \text{ u.r.s.},\ \forall n, s \tag{15}$$

$$\sum_{i\in\mathcal{I}} q_{s,n,i} - \sum_{i\in\mathcal{I}} \sum_{u\in\mathcal{U}_{n,i}} g^{\text{conv}}_{s,n,i,u} - \sum_{i\in\mathcal{I}} \sum_{e\in\mathcal{E}} g^{e}_{s,n,i} - \sum_{i\in\mathcal{I}} r^{\text{out}}_{s,n,i}$$
$$+ \sum_{i\in\mathcal{I}} r^{\text{in}}_{s,n,i} - \sum_{n'\in\mathcal{N}} T_{t(s)} B_{n,n'} v_{s,n'} = 0, \text{with } \omega_{s,n} \text{ u.r.s.},\ \forall n, s \tag{16}$$

Grid Owner and Market Clearing

$$\max_{v_{s,n}} \sum_{s\in\mathcal{S}} \sum_{n\in\mathcal{N}} P_s\, \omega_{s,n} \sum_{n'\in\mathcal{N}} T_{t(s)}\, B_{n,n'}\, v_{s,n'} \tag{11}$$

$$\text{s.t.} \sum_{n\in\mathcal{N}} H_{\ell,n} v_{s,n} \leq K_\ell\ (\overline{\mu}_{s,\ell}),\ \forall \ell, s \tag{12}$$

$$-\sum_{n\in\mathcal{N}} H_{\ell,n} v_{s,n} \leq K_\ell\ (\underline{\mu}_{s,\ell}),\ \forall \ell, s \tag{13}$$

$$S_n\, v_{s,n} = 0\ (\gamma_{s,n}),\ \forall n, s \tag{14}$$

$$v_{s,n} \text{ u.r.s.},\ \forall n, s \tag{15}$$

$$\sum_{i\in\mathcal{I}} q_{s,n,i} - \sum_{i\in\mathcal{I}} \sum_{u\in\mathcal{U}_{n,i}} g^{\text{conv}}_{s,n,i,u} - \sum_{i\in\mathcal{I}} \sum_{e\in\mathcal{E}} g^{e}_{s,n,i} - \sum_{i\in\mathcal{I}} r^{\text{out}}_{s,n,i}$$
$$+ \sum_{i\in\mathcal{I}} r^{\text{in}}_{s,n,i} - \sum_{n'\in\mathcal{N}} T_{t(s)} B_{n,n'} v_{s,n'} = 0, \text{with } \omega_{s,n} \text{ u.r.s.},\ \forall n, s \tag{16}$$

Grid Owner and Market Clearing

$$\max_{v_{s,n}} \sum_{s\in\mathcal{S}} \sum_{n\in\mathcal{N}} P_s\, \omega_{s,n} \sum_{n'\in\mathcal{N}} T_{t(s)}\, B_{n,n'}\, v_{s,n'} \tag{11}$$

$$\text{s.t.} \sum_{n\in\mathcal{N}} H_{\ell,n} v_{s,n} \leq K_\ell\ (\overline{\mu}_{s,\ell}),\ \forall \ell, s \tag{12}$$

$$-\sum_{n\in\mathcal{N}} H_{\ell,n} v_{s,n} \leq K_\ell\ (\underline{\mu}_{s,\ell}),\ \forall \ell, s \tag{13}$$

$$S_n\, v_{s,n} = 0\ (\gamma_{s,n}),\ \forall n, s \tag{14}$$

$$v_{s,n} \text{ u.r.s.},\ \forall n, s \tag{15}$$

$$\sum_{i\in\mathcal{I}} q_{s,n,i} - \sum_{i\in\mathcal{I}} \sum_{u\in\mathcal{U}_{n,i}} g^{\text{conv}}_{s,n,i,u} - \sum_{i\in\mathcal{I}} \sum_{e\in\mathcal{E}} g^{e}_{s,n,i} - \sum_{i\in\mathcal{I}} r^{\text{out}}_{s,n,i}$$
$$+ \sum_{i\in\mathcal{I}} r^{\text{in}}_{s,n,i} - \sum_{n'\in\mathcal{N}} T_{t(s)} B_{n,n'} v_{s,n'} = 0, \text{with } \omega_{s,n} \text{ u.r.s.},\ \forall n, s \tag{16}$$

Grid Owner and Market Clearing

$$\max_{v_{s,n}} \sum_{s\in\mathcal{S}} \sum_{n\in\mathcal{N}} P_s\, \omega_{s,n} \sum_{n'\in\mathcal{N}} T_{t(s)}\, B_{n,n'}\, v_{s,n'} \tag{11}$$

$$\text{s.t.} \sum_{n\in\mathcal{N}} H_{\ell,n} v_{s,n} \leq K_\ell\ (\overline{\mu}_{s,\ell}),\ \forall \ell, s \tag{12}$$

$$-\sum_{n\in\mathcal{N}} H_{\ell,n} v_{s,n} \leq K_\ell\ (\underline{\mu}_{s,\ell}),\ \forall \ell, s \tag{13}$$

$$S_n\, v_{s,n} = 0\ (\gamma_{s,n}),\ \forall n, s \tag{14}$$

$$v_{s,n} \text{ u.r.s.},\ \forall n, s \tag{15}$$

$$\sum_{i\in\mathcal{I}} q_{s,n,i} - \sum_{i\in\mathcal{I}} \sum_{u\in\mathcal{U}_{n,i}} g^{\text{conv}}_{s,n,i,u} - \sum_{i\in\mathcal{I}} \sum_{e\in\mathcal{E}} g^{e}_{s,n,i} - \sum_{i\in\mathcal{I}} r^{\text{out}}_{s,n,i}$$
$$+ \sum_{i\in\mathcal{I}} r^{\text{in}}_{s,n,i} - \sum_{n'\in\mathcal{N}} T_{t(s)} B_{n,n'} v_{s,n'} = 0, \text{ with } \omega_{s,n} \text{ u.r.s.},\ \forall n, s \tag{16}$$

Producers' KKT Conditions

$$0 \leq -P_s \left[D^{\text{int}}_{t(s),n} - D^{\text{slp}}_{t(s),n} \left(\sum_{i' \in \mathcal{I}} q_{s,n,i'} + q_{s,n,i} \right) - \omega_{s,n} \right] + \theta_{si} \perp q_{s,n,i} \geq 0, \ \forall i, n, s \tag{17}$$

$$0 \leq P_s \left(C^{\text{conv}}_{n,i,u} - \omega_{s,n} \right) + \beta^{\text{conv}}_{s,n,i,u} + \beta^{\text{up}}_{s,n,i,u} - \sum_{s' \in \mathcal{F}_s} \beta^{\text{up}}_{s',n,i,u} - \theta_{s,i} \perp g^{\text{conv}}_{s,n,i,u} \geq 0, \forall i, n, s, \forall u \in \mathcal{U}_{n,i} \tag{18}$$

$$0 \leq P_s \, C^{\text{up}}_{n,i,u} - \beta^{\text{up}}_{s,n,i,u} \perp g^{\text{up}}_{s,n,i,u} \geq 0, \ \forall i, n, s, \forall u \in \mathcal{U}_{n,i} \tag{19}$$

$$0 \leq -P_s \, \omega_{s,n} - \theta_{s,i} + \beta^{e}_{s,n,i} \perp g^{e}_{s,n,i} \geq 0, \ \forall e, i, n, s \tag{20}$$

$$0 \leq P_s \, \omega_{s,n} + \theta_{s,i} - E^{\text{in}} \, \lambda^{\text{bal}}_{s,n,i} + \lambda^{\text{in}}_{s,n,i} \perp r^{\text{in}}_{s,n,i} \geq 0, \ \forall i, n, s \tag{21}$$

$$0 \leq P_s \left(C^{\text{sto}} - \omega_{s,n} \right) - \theta_{s,i} + \lambda^{\text{bal}}_{s,n,i} + \lambda^{\text{out}}_{s,n,i} \perp r^{\text{out}}_{s,n,i} \geq 0, \ \forall i, n, s \tag{22}$$

$$0 \leq \lambda^{\text{bal}}_{s,n,i} - \sum_{s' \in \mathcal{F}_s} T_{t(s)} \cdot (1 - E^{\text{sto}}) \cdot \lambda^{\text{bal}}_{s',n,i} + \lambda^{\text{ub}}_{s,n,i} - \lambda^{\text{lb}}_{s,n,i} \perp r_{s,n,i} \geq 0, \ \forall i, n, s \tag{23}$$

$$\sum_{n \in \mathcal{N}} q_{s,n,i} - \sum_{n \in \mathcal{N}} \sum_{u \in \mathcal{U}_{n,i}} g^{\text{conv}}_{s,n,i,u} - \sum_{n \in \mathcal{N}} \sum_{e \in \mathcal{E}} g^{e}_{s,n,i} - \sum_{n \in \mathcal{N}} r^{\text{out}}_{s,n,i} + \sum_{n \in \mathcal{N}} r^{\text{in}}_{s,n,i} = 0, \ \theta_{s,i} \text{ u.r.s.}, \forall i, s \tag{24}$$

$$0 \leq T_{t(s)} \, \overline{G}^{\text{conv}}_{n,i,u} - g^{\text{conv}}_{s,n,i,u} \perp \beta^{\text{conv}}_{s,n,i,u} \geq 0, \ \forall i, n, s, \forall u \in \mathcal{U}_{n,i} \tag{25}$$

$$0 \leq -g^{\text{conv}}_{s,n,i,u} + g^{\text{conv}}_{a(s),n,i,u} + g^{\text{up}}_{s,n,i,u} \perp \beta^{\text{up}}_{s,n,i,u} \geq 0, \ \forall i, n, s, \forall u \in \mathcal{U}_{n,i} \tag{26}$$

$$g^{e}_{s,n,i} - T_{t(s)} \, A^{e}_{s,n} \, \overline{G}^{e}_{n,i} = 0, \ \beta^{e}_{s,n,i} \text{ u.r.s.}, \ \forall e, i, n, s \tag{27}$$

$$r_{s,n,i} - T_{t(s)} \, (1 - E^{\text{sto}}) \, r_{a(s),n,i} - E^{\text{in}} \, r^{\text{in}}_{s,n,i} + r^{\text{out}}_{s,n,i} = 0, \ \lambda^{\text{bal}}_{s,n,i} \text{ u.r.s.}, \ \forall i, n, s \tag{28}$$

$$0 \leq T_{t(s)} \cdot R^{\text{in}} \cdot \overline{R}_{n,i} - r^{\text{in}}_{s,n,i} \perp \lambda^{\text{in}}_{s,n,i} \geq 0, \ \forall i, n, s \tag{29}$$

$$0 \leq T_{t(s)} \cdot R^{\text{out}} \cdot \overline{R}_{n,i} - r^{\text{out}}_{s,n,i} \perp \lambda^{\text{out}}_{s,n,i} \geq 0, \ \forall i, n, s \tag{30}$$

$$0 \leq \overline{R}_{n,i} - r_{s,n,i} \perp \lambda^{\text{ub}}_{s,n,i} \geq 0, \ \forall i, n, s \tag{31}$$

$$0 \leq -\underline{R}_{n,i} + r_{s,n,i} \perp \lambda^{\text{lb}}_{s,n,i} \geq 0, \ \forall i, n, s \tag{32}$$

Producers' KKT Conditions

$$0 \leq -P_s \left[D^{\text{int}}_{t(s),n} - D^{\text{slp}}_{t(s),n} \left(\sum_{i' \in \mathcal{I}} q_{s,n,i'} + q_{s,n,i} \right) - \omega_{s,n} \right] + \theta_{si} \perp q_{s,n,i} \geq 0, \ \forall i, n, s \tag{17}$$

$$0 \leq P_s \left(C^{\text{conv}}_{n,i,u} - \omega_{s,n} \right) + \beta^{\text{conv}}_{s,n,i,u} + \beta^{\text{up}}_{s,n,i,u} - \sum_{s' \in \mathcal{F}_s} \beta^{\text{up}}_{s',n,i,u} - \theta_{s,i} \perp g^{\text{conv}}_{s,n,i,u} \geq 0, \forall i, n, s, \forall u \in \mathcal{U}_{n,i} \tag{18}$$

$$0 \leq P_s \, C^{\text{up}}_{n,i,u} - \beta^{\text{up}}_{s,n,i,u} \perp g^{\text{up}}_{s,n,i,u} \geq 0, \ \forall i, n, s, \forall u \in \mathcal{U}_{n,i} \tag{19}$$

$$0 \leq -P_s \, \omega_{s,n} - \theta_{s,i} + \beta^{e}_{s,n,i} \perp g^{e}_{s,n,i} \geq 0, \ \forall e, i, n, s \tag{20}$$

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$$\sum_{n \in \mathcal{N}} q_{s,n,i} - \sum_{n \in \mathcal{N}} \sum_{u \in \mathcal{U}_{n,i}} g^{\text{conv}}_{s,n,i,u} - \sum_{n \in \mathcal{N}} \sum_{e \in \mathcal{E}} g^{e}_{s,n,i} - \sum_{n \in \mathcal{N}} r^{\text{out}}_{s,n,i} + \sum_{n \in \mathcal{N}} r^{\text{in}}_{s,n,i} = 0, \ \theta_{s,i} \text{ u.r.s.}, \forall i, s \tag{24}$$

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$$g^{e}_{s,n,i} - T_{t(s)} \, A^{e}_{s,n} \, \overline{G}^{e}_{n,i} = 0, \ \beta^{e}_{s,n,i} \text{ u.r.s.}, \ \forall e, i, n, s \tag{27}$$

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$$0 \leq -P_s \left[D^{\text{int}}_{t(s),n} - D^{\text{slp}}_{t(s),n} \left(\sum_{i' \in \mathcal{I}} q_{s,n,i'} + q_{s,n,i} \right) - \omega_{s,n} \right] + \theta_{si} \perp q_{s,n,i} \geq 0, \ \forall i, n, s \tag{17}$$

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$$0 \leq P_s \, C^{\text{up}}_{n,i,u} - \beta^{\text{up}}_{s,n,i,u} \perp g^{\text{up}}_{s,n,i,u} \geq 0, \ \forall i, n, s, \forall u \in \mathcal{U}_{n,i} \tag{19}$$

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$$0 \leq T_{t(s)} \, \overline{G}^{\text{conv}}_{n,i,u} - g^{\text{conv}}_{s,n,i,u} \perp \beta^{\text{conv}}_{s,n,i,u} \geq 0, \ \forall i, n, s, \forall u \in \mathcal{U}_{n,i} \tag{25}$$

$$0 \leq -g^{\text{conv}}_{s,n,i,u} + g^{\text{conv}}_{a(s),n,i,u} + g^{\text{up}}_{s,n,i,u} \perp \beta^{\text{up}}_{s,n,i,u} \geq 0, \ \forall i, n, s, \forall u \in \mathcal{U}_{n,i} \tag{26}$$

$$g^{e}_{s,n,i} - T_{t(s)} \, A^{e}_{s,n} \, \overline{G}^{e}_{n,i} = 0, \ \beta^{e}_{s,n,i} \text{ u.r.s.}, \ \forall e, i, n, s \tag{27}$$

$$r_{s,n,i} - T_{t(s)} \, (1 - E^{\text{sto}}) \, r_{a(s),n,i} - E^{\text{in}} \, r^{\text{in}}_{s,n,i} + r^{\text{out}}_{s,n,i} = 0, \ \lambda^{\text{bal}}_{s,n,i} \text{ u.r.s.}, \ \forall i, n, s \tag{28}$$

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$$0 \leq -\underline{R}_{n,i} + r_{s,n,i} \perp \lambda^{\text{lb}}_{s,n,i} \geq 0, \ \forall i, n, s \tag{32}$$

Grid Owner's KKT Conditions

$$\sum_{n' \in \mathcal{N}} P_s \, T_{t(s)} \, B_{n',n} \, \omega_{s,n'} - \sum_{\ell \in \mathcal{L}} H_{\ell,n} \, \overline{\mu}_{s,\ell} + \sum_{\ell \in \mathcal{L}} H_{\ell,n} \, \underline{\mu}_{s,\ell}$$

$$- S_n \, \gamma_{s,n} = 0 \text{ with } v_{s,n} \text{ u.r.s., } \forall n, s \tag{33}$$

$$0 \leq - \sum_{n \in \mathcal{N}} H_{\ell,n} v_{s,n} + K_\ell \perp \overline{\mu}_{s,\ell} \geq 0, \; \forall \ell, s \tag{34}$$

$$0 \leq \sum_{n \in \mathcal{N}} H_{\ell,n} v_{s,n} + K_\ell \perp \underline{\mu}_{s,\ell} \geq 0, \; \forall \ell, s \tag{35}$$

$$S_n \, v_{s,n} = 0 \text{ with } \gamma_{s,n} \text{ u.r.s., } \forall n, s \tag{36}$$

Numerical Examples

Demand

Node	Reference demand (GW)
$n1$	62
$n2$	55
$n3$	2
$n4$	8
$n5$	3
$n6$	8
$n7$	3

Conventional Generation

Type	Marginal cost (€/MWh)	Ramp-up cost (€/MWh)	CO_2 emissions per unit of electricity output (kg/kWh)
$u1$ (nuclear)	10	6.7	0
$u2$ (lignite)	20	6.7	0.94
$u3$ (coal)	22	4.7	0.83
$u4$ (CCGT)	30	5.8	0.37
$u5$ (gas)	45	2.3	0.50
$u6$ (oil)	60	2.3	0.72
$u7$ (hydro)	0	6.7	0

Installed Capacity by Producer and at Node (GW)

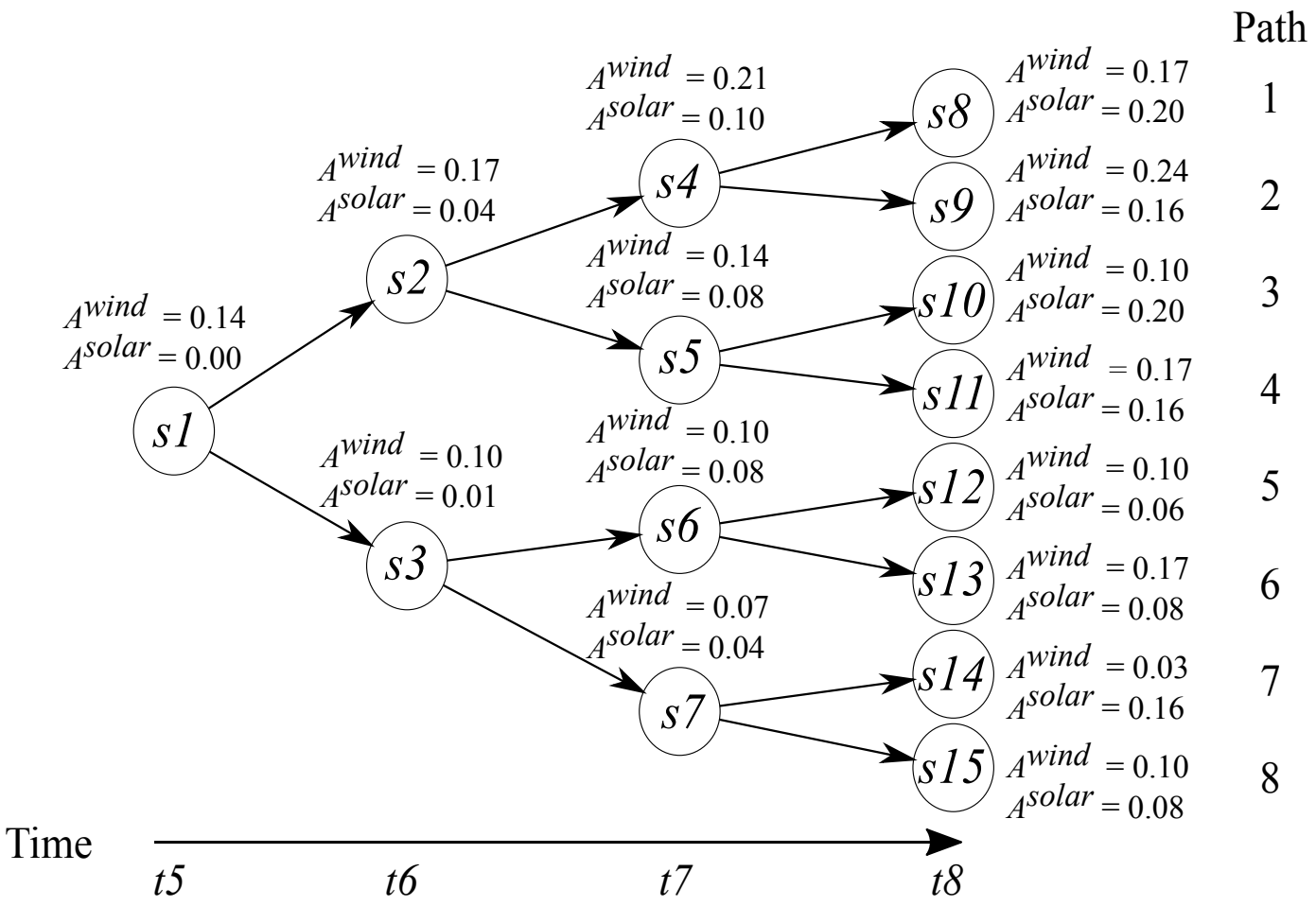
Node	Producer	$u1$	$u2$	$u3$	$u4$	$u5$	$u6$	$u7$	S	W
$n1$	E.ON	5	1	5	2	2	1	-	-	-
	RWE	4	10	5	3	3	-	0.5	-	0.5
	EnBW	3	-	4	0.5	-	0.5	0.5	-	-
	Vattenfall	-	9	2	1	1	0.5	-	-	-
	FringeD	-	-	9	4.5	4	2	1	25	28.5
$n2$	EDF	63	-	4	-	-	7	15	-	0.5
	FringeF	-	2	-	3	2	-	2	-	6.5
$n3$,	Electrabel	4	-	-	3	1	-	-	-	0.5
$n6$	FringeB	2	-	1	1	-	-	-	2	0.5
$n4$,	Electrabel	-	-	1	3	2	-	-	-	0.5
$n5$,	Essent	-	-	1	1.5	-	-	-	-	-
$n7$	Nuon	-	-	1	2	1	-	-	-	0.5
	FringeN	-	-	1	3.5	2	-	-	-	1.5
%	Available	80	85	84	89	86	86	30	See fig.	See fig.

Storage

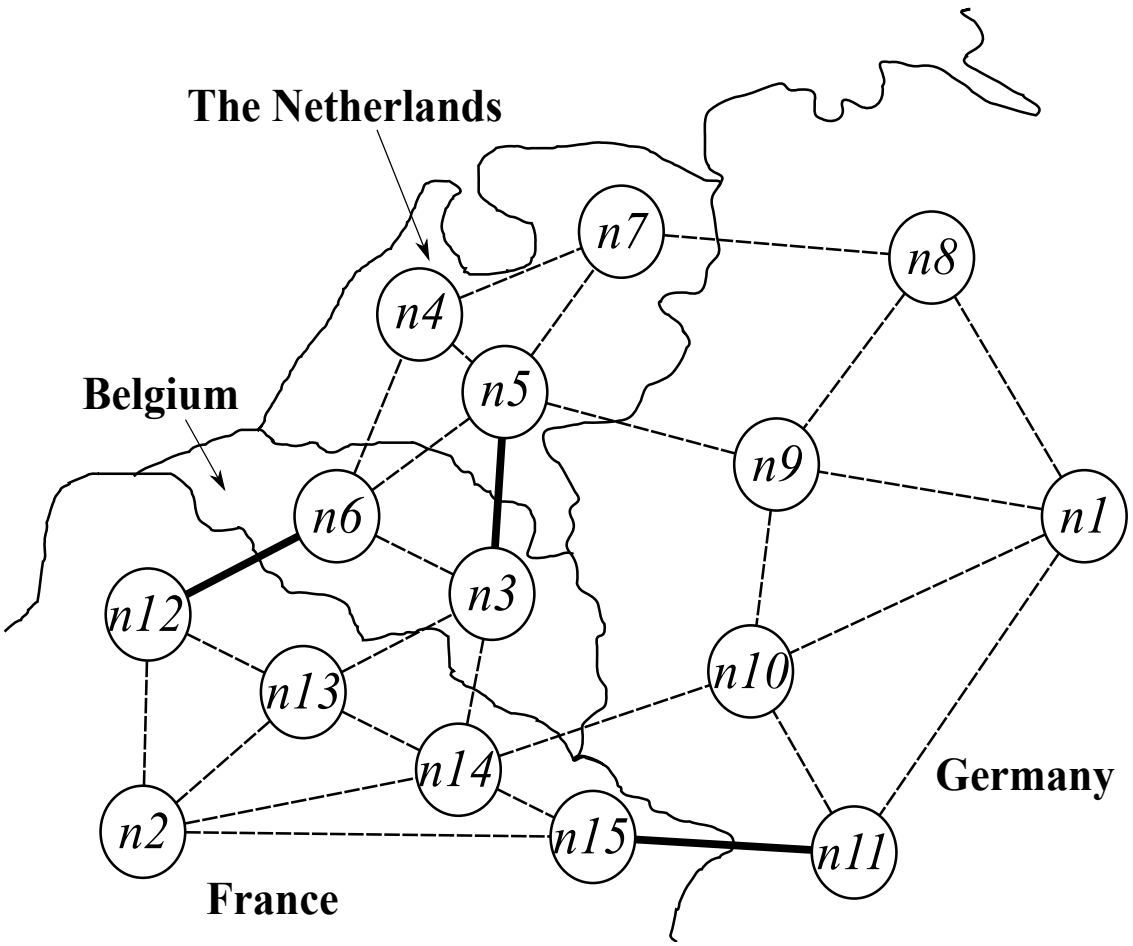
Node	Producer	Capacity in energy (GWh)
$n1$	E.ON	5
$n1$	RWE	11
$n1$	EnBW	1
$n1$	Vattenfall	16
$n1$	FringeD	3
$n2$	EDF	30
$n3$,$n6$	Electrabel	6

Parameter	Description	Value
E^{sto}	Hourly rate of storage decay (MW/MWh)	0
E^{in}	Storage input efficiency (–)	0.75
R^{in}	Hourly rate at which storage can be charged (MW/MWh)	0.16
R^{out}	Hourly rate at which storage can be discharged (MW/MWh)	0.16
R^{min}	Minimum storage share (–)	0.30

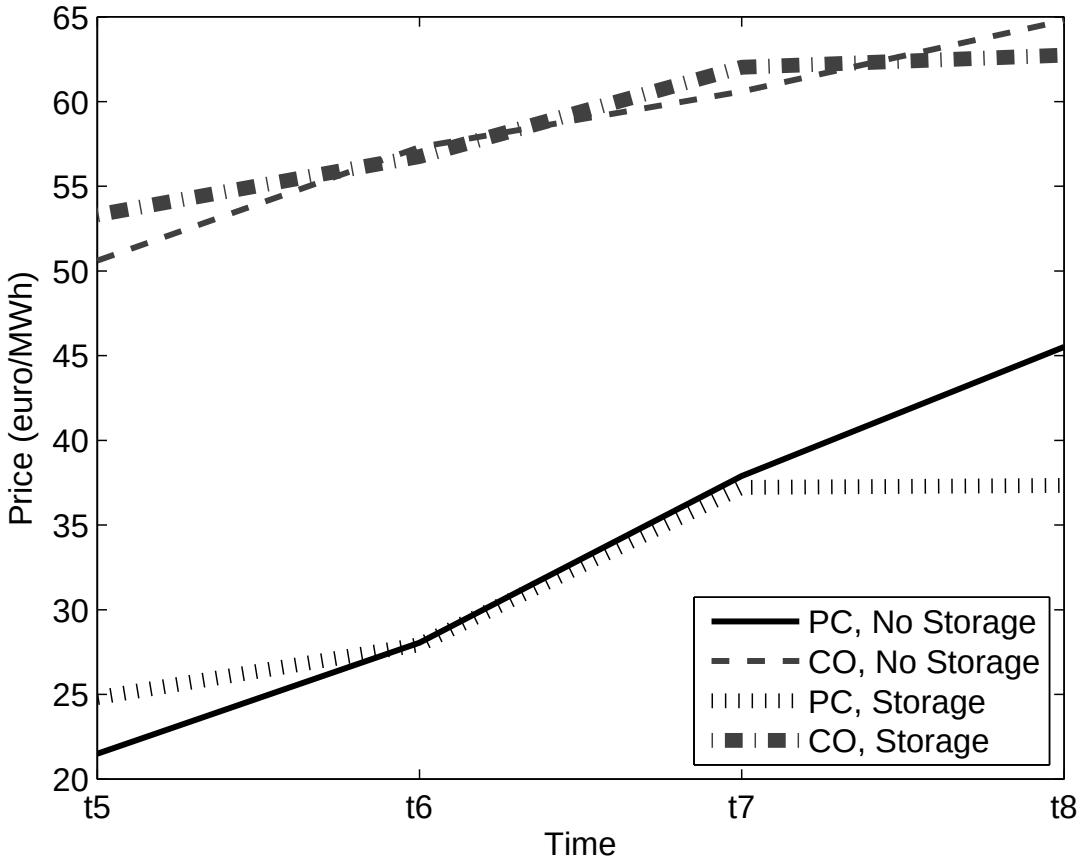
RE Generation Scenarios



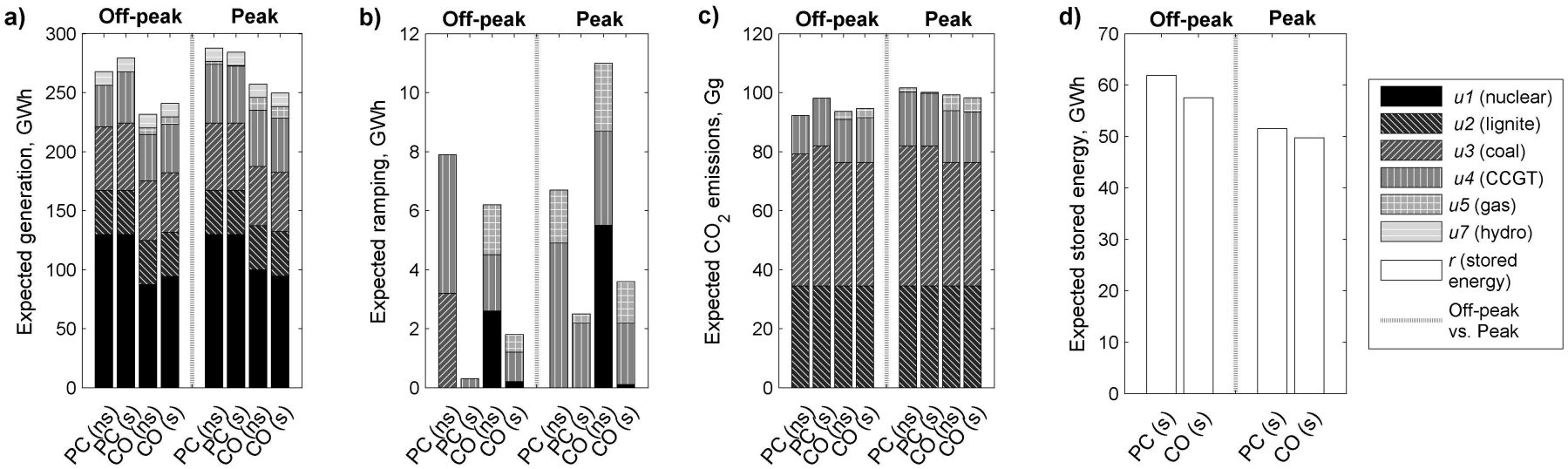
Western European Test Network (Gabriel and Leuthold, 2010)



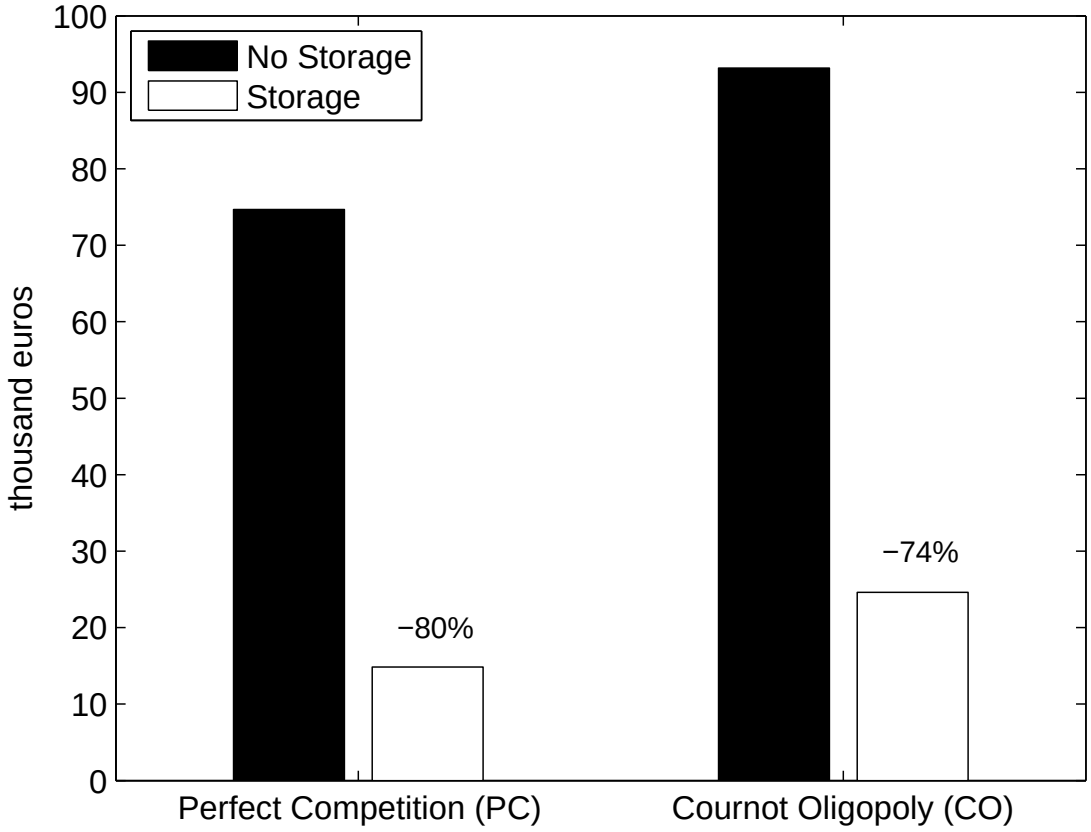
Expected Prices



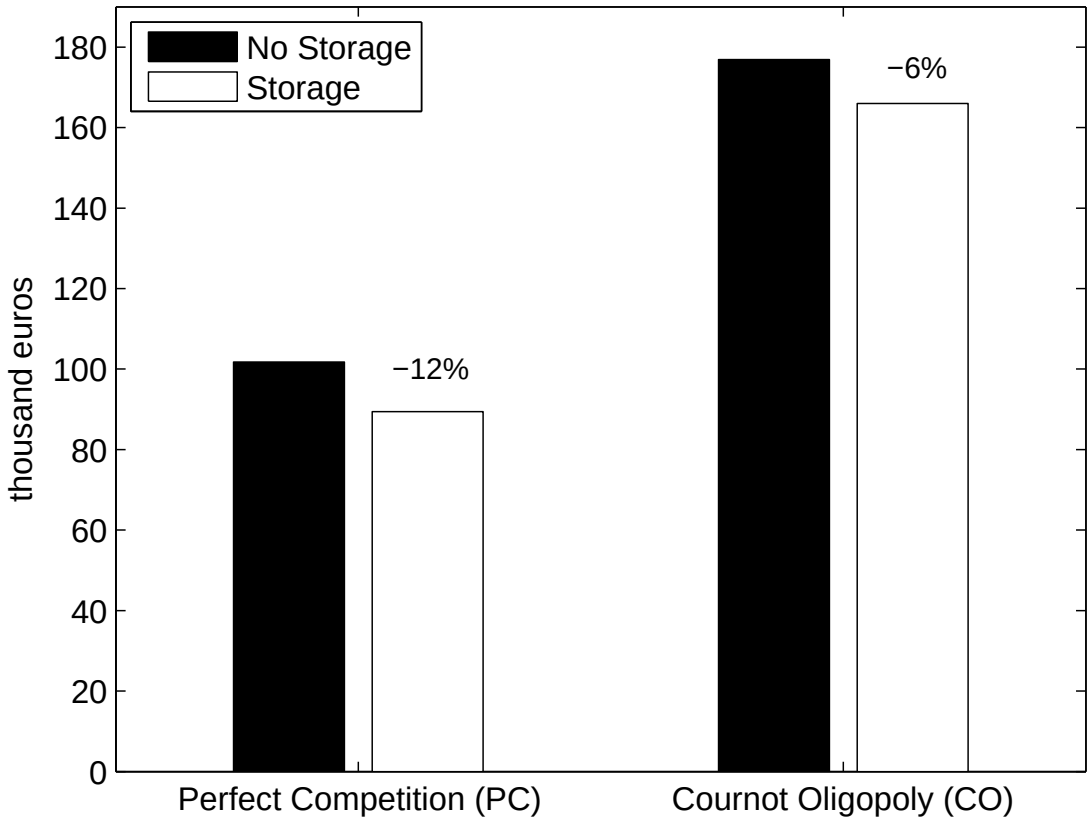
Expected Generation, Ramping, CO_2 Emissions, and Use of Storage



Effect of Storage on Expected Ramping Costs



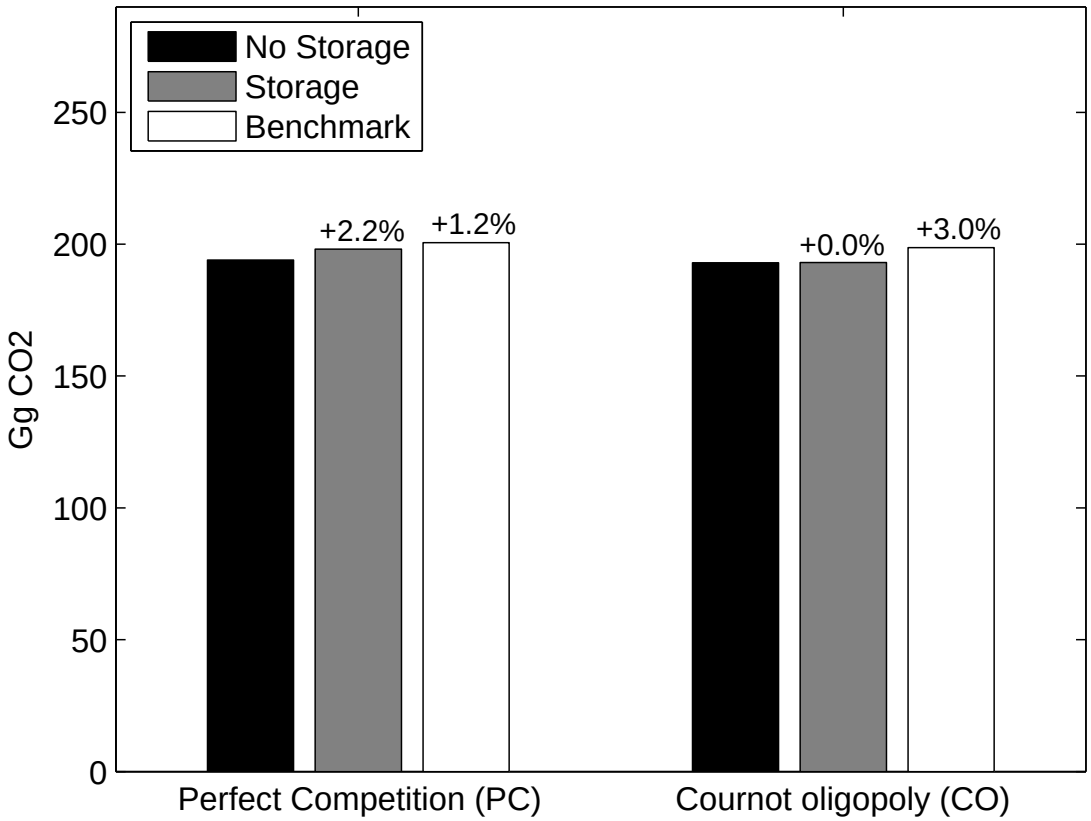
Effect of Storage on Expected Congestion Costs



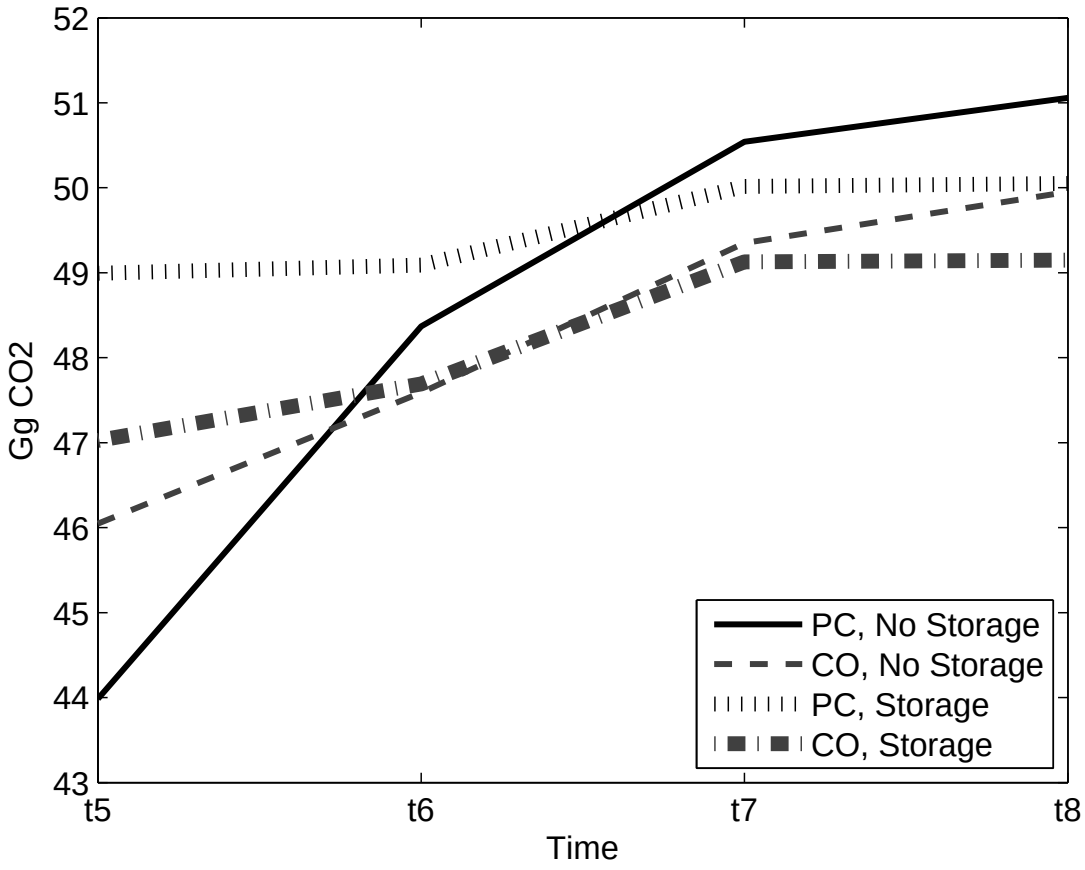
Effect of Storage on Expected Hourly Power Flows (GW)

Hour	PC No storage	PC Storage	Δ	CO No Storage	CO Storage	Δ
$t5$	15.3	13.8	-1.5	14.1	15.2	1.1
$t6$	14.4	13.4	-1.0	14.8	14.8	0.0
$t7$	14.6	14.1	-0.5	14.9	15.0	0.1
$t8$	14.1	14.7	0.6	14.7	14.4	-0.3
Σ	58.4	56.0	-2.4	58.6	59.4	0.8
$\overline{x}$	14.6	14.0	-0.6	14.6	14.8	0.2
σ	0.5	0.6	0.1	0.4	0.3	-0.1

Total Expected CO_2 Emissions



Hourly Expected CO_2 Emissions



Conclusions

Summary

- High penetration of renewables in the EU will require better congestion management, reinforcement of the transmission network, and enhanced demand response
- Energy storage is touted as a mechanism for integrating RE technologies and enhancing sustainability
- Show that many of the findings in the extant literature carry over under uncertainty and transmission constraints
- Storage can also alleviate congestion and ramping of conventional plants
- May lead to increased CO_2 emissions as a result of change in generation mix
- Impact is less severe when market power is exercised and storage used strategically
- Further policy analysis with regulation and bi-level model for optimal storage investments